TRICKY PROBLEM

Problem appears to be difficult, but can be solved easily with an idea

Find the sum of all integer numbers 'n' for which $\frac{19n+7}{7n+11}$ is an integer number.

Sol: It is not possible to examine by substituting infinitely many integers in the rational function $\frac{19n+7}{7n+11}$ to examine whether it is an integer or not. Even if you guess some integers 'n' for which given Rational number is an integer. Still we are not sure that these are The only numbers .

But how do we say that these are the only numbers? OK .Let us see the solution

Dividing numerator with the denominator we get quotient and remainder. By using division algorithm we can write given expression as $\frac{19n+7}{7n+11}=\frac{1}{7}\left(19-\frac{160}{7n+11}\right)$.

$\frac{19n+7}{7n+11}$ is an integer $\Rightarrow$ $\frac{1}{7}\left(19-\frac{160}{7n+11}\right)$ must be an integer. i.e

$\left(19-\frac{160}{7n+11}\right)$ must be a multiple of 7 $\Rightarrow$7n + 11 must be a factor of 160

I.e 7n + 11 = 1,2,4,5, 8, 10, 20, 16, 32, 40, 80, 160, -1,-2,-4,-5,- 8,-10,-20,-16,-32,-40,-80,-160 ,

$\Rightarrow \frac{160}{7n+11}=\pm 160, \pm 80, \pm 40, \pm 24, \pm 10, \pm 5, \pm 4, \pm 2, \pm 1.$

If 7n + 11 = ± 160 then $\left(19-\frac{160}{7n+11}\right)$ = 18, 169 which are not divisible by 7 .Similarly we can check that 7n + 11 = - 80, 32, -10, 4.

$\Rightarrow n \in \{3-1,-3-13\}$ .Hence their sum is -14.